International Workshop on PDEs
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TITLE & ABSTRACT

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Design of effective bulk potential for nematic liquid crystals via homogenisation

Dr. Giacomo CANEVARI
University of Verona

Nematic liquid crystals are matter in a distinguished phase, where the constituent rod-shaped molecules retain long-range orientational, but not positional, ordering. The material properties of a given nematic liquid crystal may be altered by dopants, i.e. suspended nano- to micro-particles. Even under weak anchoring conditions at the surface of the inclusions, and in the so-called "dilute regime" (i.e., when the total volume occupied by the inclusions is small), dopants can still have a significant effect; for instance, they can modify the nematic-isotropic transition temperature. In this talk, we consider a Landau-de Gennes model for a periodic suspension of small colloidal inclusions in a nematic host. By studying the homogenised limit, and proving rigorous convergence results for local minimisers, we compute the effective free energy for the doped material. In particular, we show that not only the phase transition temperature, but any coefficient of the quartic Landau-de Gennes bulk potential can be tuned. The talk is based on a joint work with Arghir D. Zarnescu (BCAM, Bilbao, Spain).

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Liouville type theorems in the stationary Navier-Stokes and related equations

Professor Dongho CHAE
Chung-Ang University

Keywords: Stationary navier-Stokes equations, Liouville type theorem
We consider the stationary Navier-Stokes equations in $\mathbb{R}^3$

$$-\Delta u + (u \cdot \nabla)u = -\nabla p, \tag{1}$$
$$\nabla \cdot u = 0. \tag{2}$$

The standard boundary condition to impose at the spatial infinity is

$$u(x) \to 0 \quad \text{as} \quad |x| \to 0. \tag{3}$$

We also assume the finiteness of the Dirichlet integral,

$$\int_{\mathbb{R}^3} |\nabla u|^2 dx < +\infty. \tag{4}$$

Obviously (u, p) with $u = 0$ and p =constant is a trivial solution to (1)-(4). A very challenging open question is if there is another nontrivial solution. This Liouville type problem is wide open, and has been actively studiedrecently in the community of mathematical fluid mechanics. The explicit statement of the problem is written in Galdi's book[**?**, Remark X. 9.4, pp. 729], where under the stronger assumption $u \in L^{\frac{9}{2}}(\mathbb{R}^3)$ he concludes $u = 0$. After that many authors deduce sufficient conditions stronger than (3) and/or (4) to obtain the Liouville type result. In this talk we review various previous results and present recent progresses in getting sufficient condition in terms of the potential functions of the velocity.

References
[1] G. P. Galdi, *An introduction to the mathematical theory of the Navier- Stokes equations: Steady-State Problems*, Springer, 2011.

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Analysis of the effective viscosity of dilute suspensions

Professor David Christophe Luc GERARD-VARET
Université Paris Diderot

We are interested in the effective viscosity generated by a large number of small rigid particles immersed in a Stokes flow. When the volume fraction ϕ of the particles is small, a first order approximation of the effective viscosity is provided by Einstein's formula : $\mu_{eff} = \mu + \frac{5}{2}\phi\mu$. We will discuss in this talk the second order approximation, for which pair interaction must be taken into account. We will show how the mathematical approach developped by S. Serfaty and co-authors on Coulomb gases can be applied, providing explicit formula. This is a joint work with M. Hillairet.

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On the Ericksen-Leslie's hyperbolic model for liquid crystals

Professor Ning JIANG
Wuhan University

The original Ericksen-Leslie's model for liquid crystals includes the inertia effect, the corresponding balance laws includes second material derivatives. This system includes a coupling of Navier-Stokes equations with a S^2 valued hyperbolic system. In this talk, we review our recent work on the well-posedness of this hyperbolic Ericksen-Leslie's liquid crystal model and the justification of the zero inertia limit to the parabolic model which has been extensively studied in the past three decades. This is a series work joint with Luo, Tang, Zarnescu, and Huang, Zhao, respectively.

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Stability of spatio-temporal periodic states of the compressible Navier-Stokes equations

Professor Yoshiyuki KAGEI
Tokyo Institute of Technology

This talk is concerned with the stability of a spatio-temporal periodic state of the compressible Navier-Stokes equations in a 2 dimensional infinite layer. It will be shown that if the Reynold and Mach numbers are sufficiently small, then the spatio-temporal periodic state is asymptotically stable under small perturbations. Furthermore, it will be proved that the asymptotic leading part of the perturbation is given by a product of a spatio-temporal periodic function and a self-similar solution of a 1 dimensional Burgers equation. The proof is given by a combination of the linearized spectral analysis and the Matsumura-Nishida energy method. The spectrum of the evolution operator is analyzed by employing the Bloch transformation and the Floquet theory. This talk is based on a joint work with Dr. Shota Enomoto and Mr. Mohamad Azlan.

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The shoreline problem for the Green-Naghdi equations
(joint work with G. Métivier)

Professor David Jean Michel LANNES
Université de Bordeaux

The Green-Naghdi equations are a nonlinear dispersive perturbation of the nonlinear shallow water equations, more precise by one order of approximation. These equations are commonly used for the simulation of coastal flows, and in particular in regions where the water depth vanishes (the shoreline). The local well-posedness of the Green-Naghdi equations (and their justification as an asymptotic model for the water waves equations) has been extensively studied, but always under the assumption that the water depth is bounded from below by a positive constant. In this talk we will see how to remove this assumption. The problem then becomes a free-boundary problem since the position of the shoreline is unknown and driven by the solution itself. For the (hyperbolic) nonlinear shallow water equation, this problem is very related to the vacuum problem for a compressible gas. The Green-Naghdi equation include additional nonlinear, dispersive and topography terms with a complex degenerate structure at the boundary. In particular, the degeneracy of the topography terms makes the problem loose its quasilinear structure and become fully nonlinear. Dispersive smoothing also degenerates and its behavior at the boundary can be described by an ODE with regular singularity. These issues require the development of new tools, some of which of independent interest such as the study of the mixed initial boundary value problem for dispersive perturbations of characteristic hyperbolic systems, elliptic regularization with respect to conormal derivatives, or general Hardy-type inequalities. Ref: D. Lannes and G. Mtivier. The shoreline problem for the one-dimensional shallow water and Green- Naghdi equations. J. Ec. polytech. Math., 5:455518, 2018.

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Rigidity of the Navier-Stokes equations

Professor Zhen LEI
Fudan University

It has been an old and challenging problem to classify bounded ancient solutions of the incompressible Navier-Stokes equations, which could play a crucial role in the study of global regularity theory. In the works (see the references), the authors made the following conjecture: *for the 3D axially symmetric Navier Stokes equations, bounded mild ancient solutions are constants*. In this article, we solve this conjecture in the case that u is periodic in z. To the best of our knowledge, this seems to be the first result on this conjecture without unverified decay conditions. It also shows that nontrivial periodic solutions are not models of possible singularities or high velocity regions. Some partial results in the non-periodic case is also given.

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Sharp Gevrey regularity for supercritical surface quasi-geostrophic equations

Professor Dong LI
The Hong Kong University of Science and Technology

I will explain some new commutator estimates of Gevrey type which settles several open questions for the dissipative quasi-geostrophic equations in the supercritical regime. Connections with some earlier (sub)-optimal regularity estimates will also be discussed.

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Entropy-bounded solutions to compressible Navier-Stokes equations with far field vacuum

Professor Jinkai LI
South China Normal University

The entropy of the ideal gas is expressed as a certain linear combination of the logarithms of the temperature and the density. Due to the singularity of the logarithmic function at zero, which may lead to the singularity of the entropy, it is difficult to analyze the uniform regularity of the entropy up to the nonvacuum-vacuum boundary. In this talk, we consider the case that the vacuum appears at the far field only, and it will be shown that the ideal gas can retain its uniform boundedness of the entropy, at least in a short time, as long as the far field vacuum is sufficiently weak in the sense that the initial density decays sufficiently slow at the far field.

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Thermal Effects in General Diffusion: An Energetic Variational Approach

Professor Chun LIU
Illinois Institute of Technology

(with Francesco De Anna , Pei Liu and Hao Wu)

Almost all biological activities involve transport and distribution of ions and charged particles in complicated biological environments. The complicated coupling and competition between different ionic solutions in various biological environments give the intricate specificity and selectivity in these systems. These systems are often associated with complicated, but specific biological relevant special biological and chemical conditions, such as the high concentration of specific species in solutions, which make most of the "ideal" assumptions in classical and conventional approaches irrelevant or unsuitable in the studies of biological problems. In the talk, I will explore the underlying mechanism governing various diffusion processes [4]. We will employ a general framework of energetic variational approaches, consisting of in particular, Onsager's Maximum Dissipation Principles [1, 2, 3], and their specific applications in biology and physiology [5]. I will discuss several extended general diffusion systems motivated by the study of ion channels and ionic solutions in biological cells. In particular, I will focus on our recent results in studying the interactions between different species, the boundary effects [8] and in some cases, the thermal effects [6, 7].

References

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[3] 458 Onsager, L. *Reciprocal Relations in Irreversible Processes. I.* Physical Review **37** (1931) 405-109.
[4] (with Arkadz Kirshtein and Mi-Ho Giga) *Variational Modeling And Complex Fluids*, Handbook of Mathematical Analysis in Mechanics of Viscous Fluids, edited by Anton Novotny and Yoshikazu Giga, Springer (2018).
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[6] (with Pei Liu and Simo Wu) *Non-Isothermal Electrokinetics: Energetic Variational Approach*, Communication in Mathematical Sciences, **16(5)** (2018)1451-1463 .
[7] (with Francesco De Anna) *Non-isothermal general Ericksen-Leslie system: derivation, analysis and thermodynamics consistency*, Archive of Rational Mechanics and Analysis, **231(2)** (2018) 637-717.
[8] (with Hao Wu) *An Energetic Variational Approach for the Cahn-Hilliard Equation with Dynamic Boundary Conditions: Derivation and Analysis* to appear in Archive of Rational Mechanics and Analysis (2018).

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On stability of physically reasonable solutions to the two-dimensional Navier-Stokes equations

Professor Yasunori MAEKAWA
Kyoto University

The flow past an obstacle is a fundamental object in fluid mechanics. In 1967 R. Finn and D. R. Smith proved the unique existence of stationary solutions, called the physically reasonable solutions, to the Navier-Stokes equations in a two-dimensional exterior domain modeling this type of flows when the Reynolds number is sufficiently small. The asymptotic behavior of their solution at spatial infinity has been studied in details and well understood by now, while its stability has remained open due to the difficulty specific to the two-dimensionality. In this talk we show that the physically reasonable solutions constructed by Finn and Smith are asymptotically stable with respect to small and well-localized initial perturbations.

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Some mathematical properties of Ferroelectric Liquid Crystals

Professor Jinhae PARK
Chungnam National University

Liquid crystal is a self-assembled media which is composed of anisotropic molecules. It has been used widely in a large variety of applications because it contains outstanding physical properties and is susceptible to electric fields. In particular, ferroelectric liquid crystals exhibit a spontaneous polarization which plays an important role in the system. In this talk, we study mathematical modeling of such a system and investigate some properties of the direction field in the application of electric fields.

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Suitable weak solutions of Beris-Edwards Q-tensor system for nematic liquid crystal flow

Professor Changyou WANG
Purdue University

In this talk, we will discuss the co-rotational Beris-Edwards system modeling the hydrodynamic motion of nematic liquid crystals in the framework of De Gennes Q-tensor theory. The system is a strong coupling between Navier-Stokes system and dissipative (parabolic like) system of Q-tensors. In dimension three, we establish both the existence and partial regularity of suitable weak solutions for such a Beris-Edwards system when the bulk function is either Landau-De Gennes potential (in R^3) or Maier-Saupe potential (in T^3). This is a joint work with Xianpeng Hu (City UHK) and Hengrong Du (Purdue).

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Convergence in power law nonlinear scalar field equations

Professor Zhi-Qiang WANG
Utah State University

We study the asymptotic behavior of ground state solutions of nonlinear scalar field equations with power non-linearity, and derive a convergence result in the vanishing case with the power tending to the borderline case. As a consequence, our result gives a direct proof of the logarithmic Sobolev inequality.

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Some studies on the Boltzmann equation without angular cutoff

Professor Tong YANG
City University of Hong Kong

After reviewing the progress on the Boltzmann equation without angular cutoff in recent years both on spatially homogeneous and inhomogeneous Boltzmann equation, I will present two results. One is about the regularizing effect of the homogeneous Boltzmann equation with Debye-Yukawa potential for measure valued solutions. Another one is about the well-posedness of perturbative solution to the inhomogeneous Boltzmann equation when the initial perturbation has only algebraic decay in the velocity variable.

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Global behaviors for Maxwell-Klein-Gordon equations with arbitrary large data

Professor Pin YU
Tsinghua University

On the three dimensional Euclidean space, for data with finite energy, it is well-known that the Maxwell-Klein-Gordon equations admit global solutions. However, the asymptotic behaviours of the solutions for the data with non-vanishing charge and arbitrary large size are unknown. It is conjectured that the solutions disperse as linear waves and enjoy the so-called peeling properties for pointwise estimates. We provide a gauge independent proof of the conjecture.

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The pointwise structure of the Green's function of a heat equation with a BV coefficient

Professor Shih-Hsien YU
National University of Singapore

A noval invention "Laplace wave train" was introduced to establish a global wave interactions to give the pointwise structure of the Green's function in the distribution sense.

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On the Vlasov-Maxwell-Boltzmann systems

Professor Huijiang ZHAO
Wuhan University

This talk is concerned with some results on the Cauchy problem of the Vlasov-Maxwell-Boltzmann system near Maxwellians.